IS EXPECTED UTILITY TOO RIGID A FRAMEWORK FOR MEAN VARIANCE THEORY?

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ABSTRACT
Mean –variance theory, as opposed to the simple analysis of moments, hereafter referred to as mean-variance analysis, can only be justified within the expected utility framework. Yet expected utility finds difficulty with higher moments such as skewness, since from an investor's point of view, intuitive arguments can be made in a favor of both negative and positive skewness. This paper investigates the flow of funds to equity mutual funds over the period 2007-2012, and not only finds evidence in favor of positive skewness, but also that the pivotal assumption of a preference for a positive first moment, may not, under extraordinary market circumstances, be as unambiguous as had previously been thought. To that effect, it may be that mean –variance-skewness analysis might find the less rigid foundation of a reference-dependent choice model, such a prospect theory, more amenable.

INTRODUCTION

The current theory of asset allocation rests on the work of Markowitz (1952a) and whilst intuitively satisfying, empirical testing has proven to be difficult. The Markowitz process involves two stages: the first, in which beliefs about the future performance of assets are generated through "observation and experience", lays the foundation for the second, in which optimal combinations of assets are formed. Yet the second stage assumes that moments and co-moments of asset distributions are known with certainty, since that is a requirement of expected utility theory. Already, we have a lot of balls in the air at once. In real world situations the risk preferences of investors cannot be directly observed, because the probability distribution of returns is as unknown to them, as our knowledge of their beliefs about them, is to us.

The problem of parameter uncertainty may also have deterred a more thorough investigation into the role higher moments may play in the asset allocation process. Indeed, the very term *mean-variance theory* suggests that they play no role at all. This, of course, can be justified by the use of quadratic utility functions that have no derivatives beyond the second, yet this *fait accompli* owes more to Alice in Wonderland than positive analysis, since one should surely prefer the phenomenon to dictate the functional form, rather than the functional form dictate the phenomenon. There are of course other specifications such as log utility which have proven appealing, and not inconsistent with Markowitz's thinking: a positive first derivative reflecting an investors' preference for higher returns, a negative second derivative describing an aversion to risk or volatility, and a positive third derivative or skew. The positive skew suggests that an investor may be willing to trade many small, below average returns, in exchange for the occasional abnormally high return, and the absence of rare, large black swan negative returns.

Skew however is problematic for expected utility since as Jean (1970) points outs, fixed income investors demonstrate a preference for negative skew, since they are willing to trade the possibility of the occasional abnormally high return, for many small, above average returns and the odd black swan. One could simply claim that two assets, one with positive skew, one with negative skew, cannot be compared, but such a seemingly reasonable compromise would itself be a violation of the assumption of the completeness of preferences.

Since both a positive or a negative sign for the third derivative can be logically justified, it is perhaps understandable that advocates of mean-variance only have dug a moat around their theory and effectively claimed at least no ambiguity about a preference for a positive first and negative second derivative – all assets remain comparable and may thus contribute to the diversification process that reduces variance or risk. The question is, should the moat have been dug in the first place? Mean-variance theory may need the framework of utility theory, but it comes at the price of stipulated signs for the first and second derivatives of the utility function and the non-existence of higher derivatives, even though arguments can be made for them being informative.

Mean-variance *analysis,* as opposed to utility laden mean-variance *theory,* may prove a more fruitful avenue of exploration. Without the constraints of a utility function, statistical data may be more freely investigated and the signs of moments more freely interpreted. This does not leave Markowitz's out in the cold, indeed the scenario where investors prefer higher returns and lower risk should probably be viewed as the *general* or *ceteris paribus* case of mean variance analysis, but not to the exclusion of all other possibilities, however unusual.

To illustrate, this paper investigates monthly investment flows to equity mutual funds over the period October 2007 to August 2012, and how these may have been influenced by the first four moments of the distribution of returns for the US equity indices. The author concedes that this is a tentative if not slightly premature venture. By convention, research involving monthly stock returns are expected to contain at least five years worth of data, the author has slightly less due to the lagging of certain variables. Secondly, the period in question was extremely volatile resulting in the risk that some results might be sample specific. Yet it is the extreme volatility of the period that makes it interesting since it might provide evidence of a mean-variance-skew-kurtosis profile that is antithetic to the general Markowitz case described above. A profile where the signs of the moments resulted in unanticipated investment flows. Thirdly, the extreme volatility of the period was not merely confined to the markets, but permeated the economy as a whole, and , at the time of writing, still does. As such, any explanatory model runs the risk of being underspecified. Finally, if expected utility cannot explain the result, what other theory can?

The paper will proceed as follows: Section I provides a brief literature review. Section II, describes the data and the multi-moment model. Section III outlines and discusses the results and the final section concludes.

LITERATURE REVIEW

The literature surrounding mean-variance theory, both theoretical and empirical, is extensive - even a cursory overview is beyond the scope of this paper. Instead this section will review the two major themes: the presence of higher moment in asset selection and the problem of parameter uncertainty.

Although the term mean-variance theory is inextricable associated with Markowitz, he did in fact give serious consideration to the issue of higher moments. In Markowitz (1952a, p.91) , he specifies that that mean-variance efficient portfolios may, under certain circumstances , be sub-optimal; in particular, when skewness, the third moment, enters into the investor's utility function. Therefore it would be incorrect to suggest that the "higher moment wheel" was discovered after Markowitz, he merely chose not to consider optimal portfolios in the presence of skewness.

The issue of higher moments has itself resulted in two literature strands. Adcock (2002) and Athayde and Flôres (2003), concentrate on establishing portfolio optimality whilst assuming parameter certainty. Others, such as Markowitz, et al (1993) have concentrated on metrics that measure downside risk such as negative semi-variance and the lower semi-third moment of mean-absolute deviation and skewness. It is somewhat surprising that the authors who concentrated on measuring downside risk did so in the face of their own conclusion that returns that were positively skewed, were preferred over those that were negatively skewed, where the downside risk is more acute. Indeed, with the exception of Jean (1970) who considered a broader class of assets such as fixed income and preferred stock, the preference for positive skew was ubiquitous.

The question of parameter uncertainty produced a more voluminous output than that of higher moments. Frost and Savarino (1988) argue that estimation error can be reduced by constraining portfolio weights. Again, working with portfolio weights, Britten-Jones (2002) suggests the use of prior densities.

Various authors have suggested a Bayesian approach to the problem. Kandel and Stambaugh (1996) examine the trade-off between stocks and cash. While Zellner and Chetty (1965) and Klein and Bawa (1976) sidestep the issue entirely by tying investor utility to expected future returns rather than sampling distribution parameters. However, Pástor (2000) and Pástor and Stambaugh (2000) link the updating process directly to the sampling distribution. Finally, Harvey, et al, (2004) use a Bayesian probability model for the joint distribution of asset returns, and demonstrate a positive difference in expected utility when higher moments are not ignored

DATA AND MODEL

Monthly flow of funds to/from US equity mutual funds was obtained from the Investment Company Institute (ICI) for the period 1/31/2007 to 9/30/2012. Monthly US equity index data for the Dow Jones Industrial Index and S&P 500 were obtained from Bloomberg. For purposes of comparison, the four moments of the data from the two indices were consistent with both the CRSP Equally and Value-Weighted indices (although the CRSP Equally-Weighted index showed a marginally greater standard deviation and skew, this was not statistically significant). The results presented in a later section were statistically equal regardless of which equity index was used.

Although the S & P 500 may be more reflective of the general level of equity prices in the US than the Dow Jones Industrial Average (since it is not price-weighted and contains more stocks); the Dow was preferred since it is arguably more immediately identifiable to the average mutual fund investor.

Sample moments were calculated in the usual manner where:

The sample mean: $\hat{\mu} = \frac{1}{T}$

The sample variance: $\hat{\sigma}^2 = \frac{1}{T}\sum_{t=t}^{T}(r_t - \hat{\mu})^2$

The sample skewness: $\hat{S} = \frac{1}{T\hat{\sigma}^3}\sum_{t=t}^{T}(r_t - \hat{\mu})^3$

And the sample kurtosis: $\hat{K} = \frac{1}{T\hat{\sigma}^4}\sum_{t=t}^{T}(r_t - \hat{\mu})^4$

The four unlagged moments were calculated using monthly observations over six-months and one-year

With the OLS model specified as follows:

$$\widehat{FOF_t} = \beta_0 + \beta_1\hat{\mu}_t + \beta_2\hat{\sigma}_t^2 + \beta_3\hat{S}_t + \beta_4\hat{K}_t + \epsilon_t$$

Where FOF_t = Flow of Funds in period ‘t’.

RESULTS

Table 1 presents the result of the regression where variance, skew and kurtosis were calculated over six-month, the results (unreported) for calculations using one year where qualitatively similar.

As expected, and consistent with mean variance theory, the coefficient for variance was negative and statistically significant. Flow of funds was negatively related to volatility, suggesting risk aversion. Consistent with other research, the coefficient for skewness was positive and statistically significant; suggesting that investors are willing to trade-off many small losses in return for the occasional abnormally large gain and the absence of large black swan losses. The fourth moment, kurtosis was not statistically significant

With respect to the first moment, six regressions were run, each lagging that variable an additional month. whilst leaving the other moments unlagged. From a one-month lag to a five-month lag, the variable was statistically insignificant, but monotonically approached statistical significance which was finally achieved when the variable was lagged six-months. However, in all case the coefficient was negative. Table 2 provides a correlation matrix for the variables, and indicates that multicollinearity is not the cause of the unanticipated sign.

Clearly the sign of the coefficient of the first moment and its delayed significance, is unambiguously inconsistent with mean variance theory. Together with the six-month lag, it suggests that over the period in question investors committed funds to the market as if they were following a lagged contrarian strategy: *If the market was “down” six-months ago and has not recovered, it’s time to buy!* or alternatively: *If the market was up six-months ago and has not sold off, it’s time to sell!*

TABLE 1.

ANALYSIS OF VARIANCE

Source	DoF	Sum Squares	Mean Square	F Value	Pr > F
Error	4	5.42E+9	1.36E+0	5.47	0.0007
Corrected Total	63	1.56E+10	2.48E+9		
	67	2.1E+10			
Root MSE	15,746.7	R-Square	0.25		
		Adj R-SQ	0.22		

PARAMETER ESTIMATES

Variable	Label	Estimate	Standard Err.	t Value	Pr > |t|
Intercept	Intercept	48,478.61	15,173.63	3.19	0.002
$\hat{\mu}$	Return	-3.56	1.20	-3.09	0.004
$\hat{\sigma}^2$	Variance	-23.66	8.13	-2.91	0.005
S	Skew	6,265.67	2,739.85	2.82	0.009
K	Kurtosis	-495.26	1,12.11	-0.33	0.74

TABLE 2.

CORRELATION MATIX

	RETURN	VARIANCE	SKEW	KURTOSIS
RETURN	1			
VARIANCE	-0.19	1		
SKEW	0.04	-0.14	1	
KURTOSIS	0.16	-0.10	-0.45	1

CONCLUSION

Notwithstanding the caveats expressed in the introduction, the tentative results obtained are doubly inconsistent with mean-variance theory. Firstly, it appears that the skewness of stock returns, particularly positive skew, influences the flow of funds to equity mutual funds. This result is consistent with past research, and held true over the extraordinary market performance during the period in question. Secondly, the negative coefficient for the lagged first moment cannot be accommodated by a mean-variance theory dependent on expected utility.

One the other hand, the idea that statistical moments can capture and describe the essence of investing has great intuitive appeal. Investors prefer higher returns to lower returns (positive first moment), Investors eschew risk (negative second moment). Investors will sustain small losses in return for the occasional large gain, if they avoid the black swan (positive third moment). Yet is it perhaps asking too much that this always be the case. The results seem to indicate that during periods of extraordinary market behavior, the signs of some moments may temporarily change indicating a temporary change in preferences.

One possible explanation might be that investors are, or can become under certain circumstance, more interested in relative wealth as opposed to absolute wealth, which would suggest that investment decisions are, or can become under certain circumstance, reference dependent. This in turn would suggest that something like Prospect theory may serve as a firmer foundation for mean-variance-(skewness) than the more rigid expected utility theory.

With respect to investing, prospect theory's greatest weakness, the identification of a reference point, may actually be its greatest strength, since there can be no better reference point for an investor than the total value of his or her investment account. Yet ironically, prospect theory, like expected utility theory requires parameter certainty, and to-date this remain elusive.

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