ENDOGENOUS BELIEF BIASES AND SYSTEMATIC ASSET PRICE FLUCTUATIONS

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ABSTRACT
Asset prices exhibit systematic fluctuations that persist over time with period of underpricing and overpricing, and such systematic fluctuations can hardly be explained by the fundamentals of the asset. A utility-based endogenous belief system is utilized to accommodate this market misbehavior. A short-memory and forward-looking representative agent has anticipatory optimism or pessimism which could affect his instantaneous utility, but at the same time anticipatory optimism or pessimism also causes the agent to bear ex-post cost due to bad decision making from irrationality. The agent balances these two incentives. The evolution of the optimal beliefs of the agent characterizes the asset price fluctuations in a relatively accurate way, given proper parameter choices.

INTRODUCTION

This paper studies systematic asset price fluctuations in an economy in which people hold endogenous belief biases. The key motivation is the observation that asset prices fluctuate around its long-term benchmark, displaying patterns of mean-reversion with periods of overpricing and underpricing. We show that this observation may be accommodated in a model that is utility-based and that allows for endogenous belief biases. This utility-based endogenous belief model captures some important features of decision making under uncertainty when people exhibit anticipatory emotions.

Our model of endogenous belief biases echoes a substantial body of psychological research that stresses the role of anticipatory feelings in decision making (Alpert and Raiffa, 1982; Weinstein, 1980; Buehler et al, 1994). Such a literature can generate two stylized facts. First, agents experience feelings of anticipation prior to the resolution of uncertainty. These feelings of anticipation can include hopelessness, confidence, anxiety, etc. Because these anticipatory feelings have a direct impact on the agents' well-being, they should be incorporated into the agents' instantaneous expected utility. Second, if agents hold biased beliefs ex-ante, they would make decisions that deviate from rational expectations. As a result, they would bear ex-post costs of basing investment decisions on biased beliefs. In our model, agents balance these two competing incentives and make a tradeoff between the ex-ante benefits of holding anticipatory feelings against paying the ex-post costs of making bad investment decisions due to biased beliefs.

The model traces its root to the seminal work of Caplin and Leahy (2001). They assume that a person's instantaneous utility is the sum of utility from that period and some function of the discounted utility in future periods. In particular, they allow time inconsistency of individual preferences in their model. They recognize that as time passes, so do anticipatory emotions, and agents' preferences may change as well.

Brunnermeier and Parker (2005) established an optimal expectations model that assumes forward-looking agents care about expected future utility flow, and hence have higher instantaneous well-being if they are optimistic about the future. The optimal expectations framework established in Brunnermeier and Parker (2005) involves a two-stage decision making process. In stage 1, agent chooses "optimally" subjective beliefs subject to the optimal actions of stage 2. In stage 2, the agent solves the portfolio allocation problem given subjective beliefs. Brunnermeier and Parker (2005)'s model addresses the inconsistency in the rational expectations assumption. Moreover, it provides discipline just as the rational expectations models: biases in beliefs are determined endogenously by the economic environment. Overall, beliefs impact instantaneous well-being directly through anticipatory emotions of the future flow utility and indirectly through their effect on portfolio allocations.

Yuan (2012) extends the optimal expectations framework by Brunnermeier and Parker (2005) and Brunnermeier et al (2007). In his paper, Yuan (2012) investigates the question about how the two-period optimal expectations model behaves if it is extended into an economy with finitely many periods. The key element in Yuan (2012)'s paper is the assumption that successive generations pass their subjective "optimal" beliefs onto their descendants and that descendants take these as their objective probabilities in order to derive their own subjective "optimal" beliefs. Yuan (2012) argues that under the no short-selling condition, there exists no stable and interior long-term optimal belief in almost all circumstances.

Yuan (2012)'s model could be appropriately applied to this paper. In particular, Yuan (2012)'s assumption about successive generations coincides with a market that is populated with short-memory traders, as evidenced by LeBaron (2002). Yuan (2012)'s paper lays a strong theoretical foundation for the issue this paper intends to investigate.

In this paper, we focus on the S&P 500 index as the representation of such a market with observable asset price fluctuations and that is populated with short-memory traders who inherit their objective probabilities from the last period. In order to model systematic asset price fluctuations, we assume existence of a representative trader and allow him to determine his beliefs endogenously by the economic environment. The economic environment is summarized in the trader's degree of risk aversion in each time period, and asset price fluctuations reflect the trader's changing beliefs.

MODEL OF ENDOGENOUS BELIEF BIASES

The model is based on Yuan (2012)'s contribution. I refer the reader to the original paper for more details.

We consider an economy where the uncertainty can be described by 2 states and short-selling is not allowed. An agent has the exponential utility function $u(c) = 1 - e^{-\alpha c}$, where $\alpha > 0$

represents the agent's degree of risk aversion. There are overlapping generations of agents, and each generation lives for two periods. For every generation, agents take their parents' optimal beliefs as objective probabilities of the world, and derive their own optimal beliefs according to the optimal expectations framework. An agent's optimal portfolio choice, $(c^*_{1,T}, c^*_{2,T})$, maximizes his expected future utility given his subjective beliefs, $(\hat{\pi}_{1,T}, \hat{\pi}_{2,T})$. Mathematically, an agent in generation T chooses optimal beliefs so as to maximize his well-being function:

$$\omega = \sum_{s=1}^{2} \hat{\pi}_{s,T}(1 - e^{-\alpha c^*_{s,T}(\hat{\pi}_T)}) + \sum_{s=1}^{2} \pi_{s,T}(1 - e^{-\alpha c^*_{s,T}(\hat{\pi}_T)})$$

where

(a) $c^*_{s,T}(\hat{\pi}_T)$ is obtained through maximizing the expected future utility function given subjective beliefs. That is,

$$\max_{(c_{1,T},c_{2,T})} [\hat{\pi}_{1,T} u(c_{1,T}) + \hat{\pi}_{2,T} u(c_{2,T})]$$

subject to the budget constraint

$$p_1 c_{1,T} + p_2 c_{2,T} = 1$$

where $c_{1,T} > 0$ and $c_{2,T} > 0$, and p_1, p_2 are the prices of the Arrow-Debreu securities yielding one unit in state 1 and 2, respectively.

(b) Objective probabilities are inherited from the previous generation:

$$\pi_{1,T} = \hat{\pi}_{1,T-1}, \pi_{2,T} = \hat{\pi}_{2,T-1}, \forall T = 2,3, ...$$

(c) The first generation's objective beliefs, $\pi_{1,1}$ and $\pi_{2,1}$, are given.

The long-term belief is generated through repeating the process as above. Notice that in Brunnermeier and Parker (2005), Brunnermeier et al (2007), and Yuan (2012), the asset prices are positively skewed due to optimistic bias. However, pessimistic bias could also be allowed in this framework and would generate negatively skewed asset prices. The combination of optimistic and pessimistic biases leads to asset price fluctuations as we observe in the actual market.

DATA AND MODEL FITTING

The datasets we use are the following: (1) historical quarterly U.S. Gross Domestic Product data from the Bureau of Economic Analysis, and (2) quarterly S&P 500 Adjusted Close Price from Yahoo! Finance. The U.S. Gross Domestic Product data are measured in billions of current dollars. Both datasets cover the period from 1950q1 to 2012q3. The time period is selected to be long enough to ensure our conclusion is robust. Complete datasets are presented in Table 1 of the Appendix. In a long-term perspective the stock market should be an indicator of the real economy. On the other hand, one of the most ubiquitous stock market valuation techniques is the discounted present value of future earnings, so the stock market at large incorporates a representative agent's anticipation, which could be either optimistic or pessimistic. Therefore, we might as well expect the stock market to be more volatile than the real economy. A look at the data confirms these intuitions. As can be seen from Figure 1 and 2, the U.S. GDP and the S&P 500 index adjusted close price have similar long-run trends, in terms of both magnitude and changes, but the stock market is much more volatile than the real economy.

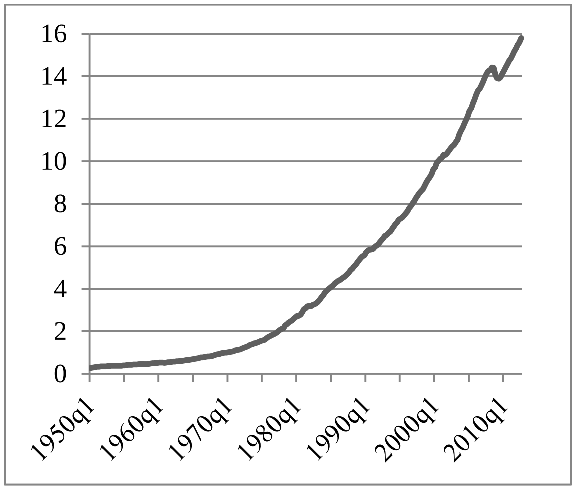

Figure 1. U.S. GDP in current dollars (billion)

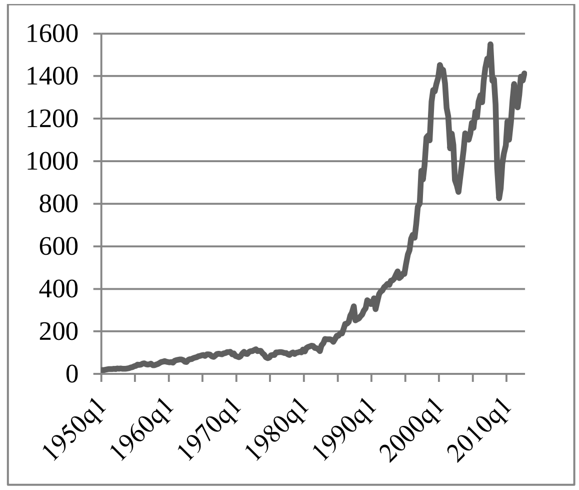

Figure 2. S&P 500 Adjusted Close Price

The juxtaposition of GDP and the S&P index gives us some useful results, but we are more interested in proposing a measure of belief of the representative agent in the market that could be incorporated into our model for further analysis. In order to do so, we calibrate our data in the following procedure. First, we set both data sets' 1950q1 data point to unity, and normalize all other data accordingly. Then we divide the normalized S&P 500 data by the normalized GDP data to get a time series of ratios. We also calculate the average of this time series. Lastly, we further normalize the time series of ratios so that the average is set at 0.5. Noticing that the distance from the average to the maximum of the time series is larger than the distance from the average to the minimum, we set max to be 1 and set average minus the distance from the average to the maximum to be 0. Hence we get a normalized time series dataset that is bounded up by 1 and bounded below by 0, with average being 0.5. The normalized time series is the last column of Table 1. Figure 3 is a plot of the dataset.

Why is the normalized time series, plotted in Figure 3, a good representation of the agent's subjective beliefs? There are three key aspects to consider. First, dividing the normalized S&P 500 data by the normalized GDP data extracts information about the agent's current feelings and anticipation about the future, which is either optimistic or pessimistic. Second, normalizing the time series is necessary because beliefs must be bounded within 0 and 1. Third, setting the long-term average to be 0.5 is a direct implication of Yuan (2012)'s result. In Yuan (2012), the only case where there is stable and interior optimal belief is when an investor starts off being unbiased and the price ratio of the Arrow-Debreu securities in the two states is 1. In this paper, if we impose the requirement that the long-term average ratio is normalized to 0.5, then an agent could hypothetically starts of at 0.5 and holds this belief forever.

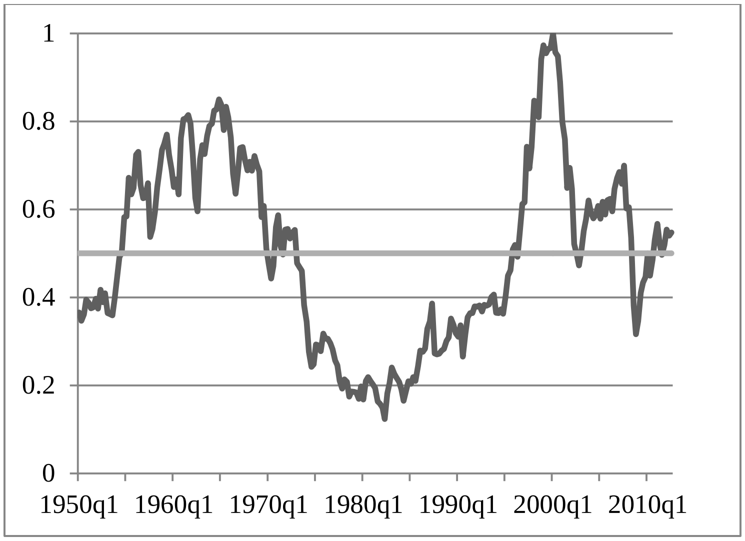

Figure 3

Now that we have the calibrated data, we can set model parameters to check if this model is a good summary of the actual market. Notice that the economic environment is completely summarized in the agent's degree of risk aversion in each time period, as aforementioned. In addition, the actual market has the following general trend: (i) upward from 1950 to 1965, (ii) then downward from 1965 to 1985; (iii) upward again from 1985 to 2000, (iv) then downward from 2000 to 2010. To fit the model, we choose 5 years to be one period (generation). We assume that the representative agent's degree of risk aversion is sticky in each of the four general trend periods. So the agent has the same degree of risk aversion in one general trend period, and updates his beliefs every five years.

By setting the price ratio of Arrow-Debreu securities in the two states to be 4.5, and the agent's degree of risk aversion large enough, we obtain the predicted values of normalized beliefs, updated every five years. The predicted values are compared to the actual values in Table 2.

An illustration of the prediction is shown in Figure 4. We can see that our model very accurately depicts the overall trends of the market misbehavior. The stock market is a noisy system with numerous different players, and the power of our model is that it extracts of the information related to expectations and that we only need to select one single parameter to visualize the long-term trend of the asset prices.

	Actual	Predicted
1950q1	0.365487	0.181
1955q1	0.583888	0.172
1960q1	0.650718	0.135
1965q1	0.838212	0.800
1970q1	0.47751	0.834
1975q1	0.292987	0.755
1980q1	0.168022	0.160
1985q1	0.203081	0.143
1990q1	0.310492	0.803
1995q1	0.404406	0.968
2000q1	1	0.995
2005q1	0.578587	1.000
2010q1	0.500148	0.480

Table 2. Actual data compared to model predictions

The result can be vastly refined in a number of ways. First, in this paper the agent updates his beliefs every five years, whereas in the actual market people update their beliefs more frequently. So a shorter time interval could take into account more information about people's changing beliefs and results would improve. In addition, a better parameter choice may be proposed.

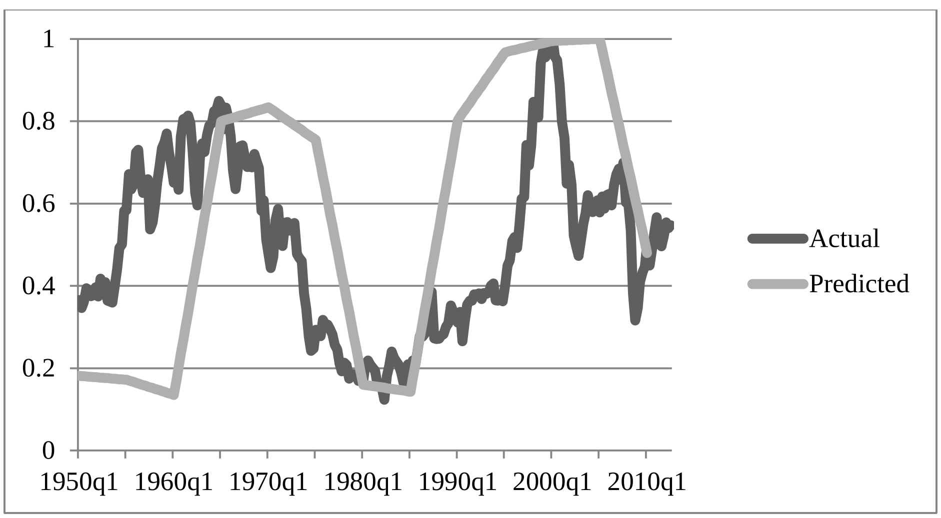

Figure 4

CONCLUSION

This paper studies systematic asset price fluctuations in an endogenous beliefs model. A representative agent is either optimistic or pessimistic and his anticipatory emotions drive the asset price fluctuations. Actual data are obtained to test the validity of the model. We find out that by choosing one single parameter the long-term trend of the market behavior is characterized in a relatively accurate way.

Further research could focus on the following. A more systematic approach needs to be developed to give guidance on predicting when the agent is optimistic and when he is pessimistic. With this information, the model would be much more useful for prediction purposes. Moreover, in this model the agent is assumed to have exponential utility function, following Yuan (2012). This utility function displays constant relative risk aversion. Further research could discuss whether another utility function might be a better choice for the particular system, or investigate whether results are robust to different utility function choices. Lastly, from a theoretical standpoint, future research could consider generalizing the model to an economy with finitely many states of the world.

DATA APPENDIX

Table 1

	GDP	S&P500	Normalized Ratio
1950q1	275.2	17.96	0.365487
1950q2	284.5	17.84	0.346262
1950q3	301.9	19.53	0.361188
1950q4	313.3	21.66	0.394633
1951q1	329.0	22.43	0.387419
1951q2	336.6	22.4	0.375165
1951q3	343.5	22.94	0.376936
1951q4	347.9	24.14	0.396534
1952q1	351.2	23.32	0.37406
1952q2	352.1	25.4	0.417233
1952q3	358.5	24.52	0.389073
1952q4	371.4	26.38	0.40888
1953q1	378.4	24.62	0.363996
1953q2	382.0	24.75	0.361943
1953q3	381.1	24.54	0.358948
1953q4	375.9	26.08	0.396477
1954q1	375.2	28.26	0.44117
1954q2	376.0	30.88	0.492395
1954q3	380.8	31.68	0.500413
1954q4	389.4	36.63	0.582238
1955q1	402.6	37.96	0.583888
1955q2	410.9	43.52	0.671372
1955q3	419.4	42.34	0.634051
1955q4	426.0	43.82	0.648423
1956q1	428.3	48.38	0.724377
1956q2	434.2	49.39	0.730331
1956q3	439.2	45.58	0.655313
1956q4	448.1	44.72	0.625363
1957q1	457.2	45.74	0.627203

1957q2	459.2	47.91	0.659482
1957q3	466.4	41.06	0.536852
1957q4	461.5	41.7	0.55432
1958q1	453.9	43.44	0.594548
1958q2	458.0	47.19	0.64971
1958q3	471.7	51.33	0.693233
1958q4	485.0	55.45	0.734699
1959q1	495.5	57.59	0.748966
1959q2	508.5	60.51	0.769816
1959q3	509.3	57.52	0.724235
1959q4	513.2	55.61	0.689773
1960q1	527.0	54.37	0.650718
1960q2	526.2	55.51	0.6682
1960q3	529.0	53.39	0.633844
1960q4	523.7	61.78	0.762075
1961q1	528.0	65.31	0.805152
1961q2	539.0	66.76	0.8064
1961q3	549.5	68.62	0.814061
1961q4	562.6	68.84	0.795124
1962q1	576.1	65.24	0.726529
1962q2	583.2	58.23	0.625713
1962q3	590.0	56.52	0.595246
1962q4	593.3	66.2	0.714002
1963q1	602.5	69.8	0.746141
1963q2	611.2	69.13	0.725485
1963q3	623.9	74.01	0.767015
1963q4	633.5	77.04	0.789478
1964q1	649.6	79.46	0.79483
1964q2	658.9	83.18	0.824321
1964q3	670.5	84.86	0.82674
1964q4	675.6	87.56	0.849622
1965q1	695.7	89.11	0.838212
1965q2	708.1	85.25	0.780317
1965q3	725.2	92.42	0.83335
1965q4	747.5	92.88	0.809373
1966q1	770.8	91.06	0.763345
1966q2	779.9	83.6	0.680999
1966q3	793.1	80.2	0.635318
1966q4	806.9	86.61	0.682079
1967q1	817.8	94.01	0.7394
1967q2	822.3	94.75	0.741438
1967q3	837.0	93.3	0.713176
1967q4	852.7	92.24	0.688379
1968q1	879.8	97.46	0.707951
1968q2	904.1	97.74	0.687879
1968q3	919.3	103.41	0.720838
1968q4	936.2	103.01	0.702344
1969q1	960.9	103.69	0.686387
1969q2	976.1	91.83	0.582318
1969q3	996.3	97.12	0.607918
1969q4	1,004.5	85.02	0.511292
1970q1	1,017.1	81.52	0.47751
1970q2	1,033.1	78.05	0.442897
1970q3	1,050.5	83.25	0.470727
1970q4	1,052.7	95.88	0.559757
1971q1	1,098.1	103.95	0.58672
1971q2	1,118.8	95.58	0.517249

1971q3	1,139.1	94.23	0.496876
1971q4	1,151.4	103.94	0.553682
1972q1	1,190.1	107.67	0.555177
1972q2	1,225.6	107.39	0.53374
1972q3	1,249.3	111.58	0.546468
1972q4	1,286.6	116.03	0.55301
1973q1	1,335.1	106.97	0.477298
1973q2	1,371.5	108.22	0.468156
1973q3	1,390.7	108.29	0.460338
1973q4	1,431.8	96.57	0.381928
1974q1	1,446.5	90.31	0.344207
1974q2	1,484.8	79.31	0.276345
1974q3	1,513.7	73.9	0.241779
1974q4	1,552.8	76.98	0.247454
1975q1	1,569.4	87.3	0.292987
1975q2	1,605.0	88.75	0.290501
1975q3	1,662.4	89.04	0.277448
1975q4	1,713.9	100.86	0.31723
1976q1	1,771.9	101.64	0.306048
1976q2	1,804.2	103.44	0.305828
1976q3	1,837.7	102.9	0.295753
1976q4	1,884.5	102.03	0.281816
1977q1	1,938.5	98.44	0.256533
1977q2	2,005.2	98.85	0.245361
1977q3	2,066.0	92.34	0.210736
1977q4	2,110.8	89.25	0.192583
1978q1	2,149.1	96.83	0.213452
1978q2	2,274.7	100.68	0.207468
1978q3	2,335.2	93.15	0.174577
1978q4	2,416.0	99.93	0.185654
1979q1	2,463.3	101.76	0.185268
1979q2	2,526.4	103.81	0.18361
1979q3	2,599.7	101.82	0.169133
1979q4	2,659.4	114.16	0.197432
1980q1	2,724.1	106.29	0.168022
1980q2	2,728.0	121.67	0.210024
1980q3	2,785.2	127.47	0.218801
1980q4	2,915.3	129.55	0.208801
1981q1	3,051.4	132.81	0.201926
1981q2	3,084.3	130.92	0.193822
1981q3	3,177.0	121.89	0.163116
1981q4	3,194.7	120.4	0.158007
1982q1	3,184.9	116.44	0.149524
1982q2	3,240.9	107.09	0.123063
1982q3	3,274.4	133.72	0.181714
1982q4	3,312.5	145.3	0.204483
1983q1	3,381.0	164.43	0.24037
1983q2	3,482.2	162.56	0.225695
1983q3	3,587.1	163.55	0.217499
1983q4	3,688.1	163.41	0.207818
1984q1	3,807.4	160.05	0.190732
1984q2	3,906.3	150.66	0.164636
1984q3	3,976.0	166.09	0.18875
1984q4	4,034.0	179.63	0.209486
1985q1	4,117.2	179.83	0.203081
1985q2	4,175.7	190.92	0.21846
1985q3	4,258.3	189.82	0.209843

1985q4	4,318.7	211.78	0.243413
1986q1	4,382.4	235.52	0.278811
1986q2	4,423.2	236.12	0.276101
1986q3	4,491.3	243.98	0.283179
1986q4	4,543.3	274.08	0.328351
1987q1	4,611.1	288.36	0.344979
1987q2	4,686.7	318.66	0.386035
1987q3	4,764.5	251.79	0.272075
1987q4	4,883.1	257.07	0.270553
1988q1	4,948.6	261.33	0.271788
1988q2	5,059.3	272.02	0.278992
1988q3	5,142.8	278.97	0.282592
1988q4	5,251.0	297.47	0.300691
1989q1	5,360.3	309.64	0.309083
1989q2	5,453.6	346.08	0.351924
1989q3	5,532.9	340.36	0.337301
1989q4	5,581.7	329.08	0.318048
1990q1	5,708.1	330.8	0.310492
1990q2	5,797.4	356.15	0.336677
1990q3	5,850.6	304	0.265404
1990q4	5,846.0	343.93	0.317106
1991q1	5,880.2	375.34	0.354724
1991q2	5,962.0	387.81	0.363872
1991q3	6,033.7	392.45	0.363842
1991q4	6,092.5	408.78	0.379287
1992q1	6,190.7	414.95	0.378778
1992q2	6,295.2	424.21	0.381474
1992q3	6,389.7	418.68	0.367463
1992q4	6,493.6	438.78	0.382865
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1994q3	7,131.8	472.35	0.372785
1994q4	7,248.2	470.42	0.362779
1995q1	7,307.7	514.71	0.404406
1995q2	7,355.8	562.06	0.449376
1995q3	7,452.5	581.5	0.461544
1995q4	7,542.5	636.02	0.508927
1996q1	7,638.2	654.17	0.518857
1996q2	7,800.0	639.95	0.491772
1996q3	7,892.7	705.27	0.546793
1996q4	8,023.0	786.16	0.611737
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1997q3	8,409.9	914.62	0.692752
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1998q2	8,698.6	1120.67	0.843829
1998q3	8,847.2	1098.67	0.808836
1998q4	9,027.5	1279.64	0.941013
1999q1	9,148.6	1335.18	0.972574
1999q2	9,252.6	1328.72	0.954977
1999q3	9,405.1	1362.93	0.964826
1999q4	9,607.7	1394.46	0.966526

2000q1	9,709.5	1452.43	1
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2002q2	10,601.9	911.62	0.52143
2002q3	10,701.7	885.76	0.497214
2002q4	10,766.9	855.7	0.472436
2003q1	10,887.4	916.92	0.508127
2003q2	11,011.6	990.31	0.551128
2003q3	11,255.1	1050.71	0.576868
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2004q1	11,589.9	1107.3	0.593316
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2005q1	12,361.8	1156.85	0.578587
2005q2	12,500.0	1234.18	0.617351
2005q3	12,728.6	1207.01	0.587947
2005q4	12,901.4	1280.08	0.621007
2006q1	13,161.4	1310.61	0.623713
2006q2	13,330.4	1276.66	0.59505
2006q3	13,432.8	1377.94	0.646289
2006q4	13,584.2	1438.24	0.671087
2007q1	13,758.5	1482.37	0.685129
2007q2	13,976.8	1455.27	0.657878
2007q3	14,126.2	1549.38	0.69972
2007q4	14,253.2	1378.55	0.602184
2008q1	14,273.9	1385.59	0.604839
2008q2	14,415.5	1267.38	0.535963
2008q3	14,395.1	968.75	0.380804
2008q4	14,081.7	825.88	0.315732
2009q1	13,923.4	872.81	0.346111
2009q2	13,885.4	987.48	0.409541
2009q3	13,952.2	1036.19	0.433249
2009q4	14,133.6	1073.87	0.446137
2010q1	14,270.3	1186.69	0.500148
2010q2	14,413.5	1101.6	0.449511
2010q3	14,576.0	1183.26	0.485254
2010q4	14,735.9	1286.12	0.531148
2011q1	14,814.9	1363.61	0.567003
2011q2	15,003.6	1292.28	0.52252
2011q3	15,163.2	1253.3	0.496356
2011q4	15,321.0	1312.41	0.51898
2012q1	15,478.3	1397.91	0.553994
2012q2	15,585.6	1379.32	0.54034
2012q3	15,797.4	1412.16	0.547054

REFERENCES

Alpert, Marc and Howard Raiffa (1982). "A Progress Report on the Training of Probability Assessors." in Daniel Kahneman, Paul Slovic and Amos Tversky, eds., *Judgment under uncertainty: Heuristics and biases,* Cambridge University Press, 294-305.

Brunnermeier, Markus K. and Jonathan A. Parker (2005). "Optimal Expectations." *American Economic Review*, Volume 95, Number 4, 1092-1118.

Brunnermeier, Markus K., Christian Gollier, and Jonathan A. Parker (2007). "Optimal Beliefs, Asset Prices, and the Preference for Skewed Returns." *American Economic Review*, Volume 97, Number 2, 159-165.

Buehler, Roger, Dale Griffin, and Michael Ross (1994). "Exploring the 'Planning Fallacy': Why People Underestimate Their Task Completion Time." *Journal of Personality and Social Psychology*, Volume 67, Number 3, 366-381.

Caplin, Andrew and John Leahy (2001). "Psychological Expected Utility Theory and Anticipatory Feelings." *Quarterly Journal of Economics*, Volume 116, Number 1, 55-79.

LeBaron, Blake (2002). "Short-Memory Traders and Their Impact on Group Learning in Financial Markets." *Proceedings of the National Academy of Sciences of the United States of America*, Volume 99, Suppl 3, 7201-7206.

Weinstein, Neil D. (1980). "Unrealistic Optimism About Future Life Events." *Journal Personality and Social Psychology*, Volume 39, Number 5, 806-820.

Yuan, Yue (2012). "Optimal Beliefs in the Long Run: An Overlapping Generations Perspective." *Economics Letters*, Volume 117, Number 2, 525-527.